

# Molar Volume, Coefficient of Thermal Expansion, and Related Properties of Liquid $\mathrm{He}^{4}$ under Pressure* 

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#### Abstract

From measurements of the dielectric constant, we have determined the molar volume of liquid $\mathrm{He}^{4}$ along various isobars in the region 1.25 to $4.2^{\circ} \mathrm{K}$ and 0.5 to 28 atm . Tables of molar volume, thermal expansion coefficient $\alpha_{P}$, entropy of compression, and isobaric change in the isothermal compressibility are presented for this region. Values of $T_{\lambda}, P_{\lambda}$, and $V_{\lambda}$ are presented. Particular emphasis was put on the form of the expansion coefficient near the $\lambda$ transition. We have found that the $\alpha_{P}$ singularity is logarithmic over the range of $2 \times 10^{-5}$ to $10^{-2}{ }^{\circ} \mathrm{K}$ displacement from the transition, and from the data that lead to this conclusion, we have been able to calculate various other properties of the fluid near the transition. These results are compared with those of other experiments wherever possible.


## I. INTRODUCTION

TJE have made a detailed investigation of the molar volume of liquid $\mathrm{He}^{4}$ along various isobars between 1.25 and $4.2^{\circ} \mathrm{K}$, and from the results of this investigation we have obtained the thermal expansion coofficient, $\alpha_{P}=1 / V(\partial V / \partial T)_{P}$, and thermodynamically related properties. In this study we have put particular cmphasis on the region near the $\lambda$ transition, and on the behavior of $\alpha_{P}$ as $\left|T-T_{\lambda}\right|$ decreases to $10^{-50} \mathrm{~K}$. Similar research has been done by several workers ${ }^{1-4}$ along the vaporization curve with considerable resolution near the $\lambda$ point, and $\alpha_{P}$ has been obtained by Grilly and Mills ${ }^{5}$ along certain isobars near the melting curve but with limited resolution near the $\lambda$ transition. We have improved on this work by a factor of 10 or more in temperature resolution, and have made the first systematic study of the variation of $\alpha_{P}$ with pressure in this temperature range.
Molar volumes are of fundamental importance in obtaining an equation of state for $\mathrm{He}^{4}$. The fact that the molar volume decreases with increasing temperature $\left(\alpha_{P}<0\right)$ over the entire range of our data below the $\lambda$ line and, for part of the range above it, is most unusual and therefore worth studying in detail. In addition, such an isobaric study yields a particularly sensitive way of calculating changes in the isothermal compressibility, $\Delta k_{T}$, with temperature, and of calculating the entropy of compression. Our results of $V$ and of $\alpha_{P}$, and our calculations of $\Delta k_{T}=k_{T}(T)-k_{T}\left(T_{0}\right)$ and $S_{\text {comp }}=S(T, P)$ $-S\left(T, P_{\mathrm{SVP}}\right)$ are presented in the first part of the results section, and are compared with existing data where possible.
Our study was undertaken primarily to determine the nature of the $\lambda$ transition at elevated pressures. This

[^0]requires quite high resolution in the independent variables, since previous studies near the transition have all shown that the various properties change rapidly. Fairbank, Buckingham, and Kellers ${ }^{6,7}$ have shown that the singularity in the specific heat of the liquid under its saturated vapor is logarithmic over a wide range. Thermodynamic relations ${ }^{7}$ indicate that $\alpha_{P}$ and $C_{P}$ ought to display the same limiting behavior at $T_{\lambda}$. The thermal expansion $\alpha_{P}$ is more readily measurable at elevated pressures because the experiment, unlike that of $C_{P}$, does not involve thermal isolation from a bath. This isolation is difficult to achieve because of the heat leak through the pressure-transmitting capillary. Hence it was decided to make a systematic study of $\alpha_{P}$ close to the transition line.

A by-product of this research is a new determination of the $\lambda$ line in which particular care has been taken to determine the limiting slope at the lower triple point, and we present the results of this determination expressed as $P_{\lambda}(T)$ and $V_{\lambda}(T)$. We present our expansioncoefficient results close to the transition at the various pressures where we have taken high-resolution data. From these results we have calculated by various means the specific heat $C_{P}$, the compressibility coefficient $k_{T}$, and the pressure coefficient $(\partial P / \partial T)_{V}$. All of these data are presented in the second section of results along with a discussion of internal consistency and a comparison with the results of previous workers. In general the agreement is quite good so that our results may be taken as giving a valid description of the shape of the volume surface near the $\lambda$ line. Some of the results have already been reported in more concise form elsewhere. ${ }^{8}$

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